Symmetric functions and Hopf algebras Usage and design in MuPAD-Combinat

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Prehistory

- Schützenberger, Thibon, ...
- ▶ Programs in Pascal, C, ...
- ► Not really user friendly
- Hard to maintain, not really distributed

And then came Maple

- Ease of use
- Higher level programming language

Software

- SF (Stembridge)
- ► ACE (Veigneau, Lascoux, Thibon, Ung, ...)
- \blacktriangleright μ -EC (Prosper, Carré)

Design

- ► Data-structure : expressions
- Operators: expansion, bases change, scalar product, inner product, plethysm, ...
- Hall Littlewood, Mac Donald, NCSF, QSym, ...

Advantages

- Flexibility
- Easy to use (at least apparently?)
- Widely available platform

Drawbacks

- Sloppy data structure (expression parsing)
- ▶ Non trivial coefficient rings ($\mathbb{Z}/2\mathbb{Z}$, degree 1 elements, ...)?
- Non commutative Hopf algebras?
- Naming conflicts
- Speed?
- Maple

White book for MuPAD-Combinat

Goals

- Experimentation tool in the study of (Hopf) algebras
- ► Ease of use, expressiveness, flexibility, extensibility
- ► Speed?
- ▶ Managing 30+ algebras, algebras with 10+ bases
- Code sharing, long term maintenance

White book for MuPAD-Combinat

Design decisions

- Object orientation
- ► MuPAD platform
- ▶ Reuse of existing software (Symmetrica, 1rcalc, ...)
- Open source
- ► Core development by "senior" researchers
- Decentralized development

The MuPAD platform

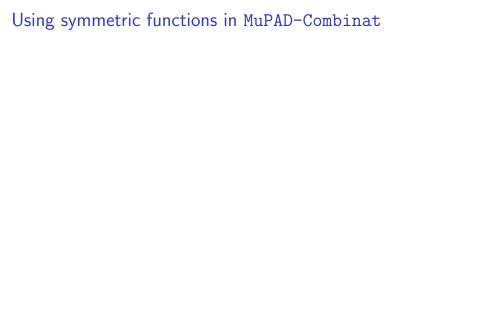
- Developed by Padderborn / Sciface since 1980's
- Not open source (bummer, bummer, bummer)
- Fairly open
- Reasonably priced, fairly widespread

Reasonable programming language

- Object oriented (encapsulation, Domains/Axioms/Categories, reflection)
- Functional programming (closures, ...)
- Dynamic modules (C++ integration)
- Very (too?) flexible
- But special purpose

MuPAD-Combinat figures

- ▶ 8 developers, 20 contributers, 25+ research articles
- Official MuPAD library since 2002, NSF Sponsored
- ▶ 7 years, 10 official releases, 6 stable ones
- GNU/Linux, MacOS X, Windows, Zaurus
- ▶ 100000 lines of MuPAD, 15000 lines of C++
- ▶ 26000 lines of tests, 575 pages of doc
- ▶ In 2005 : 1500 messages on the mailing list, 5000 visits of the web page and 400 downloads.
- ▶ Integrated software : μ -EC, CS, PerMuVAR, Symmetrica, Ircalc, Nauty, rigged configuration kernel
- ► How many users?



The Hopf algebra framework

Building bricks

- Combinatorial classes
- Free-modules
- Category hierarchy
- Overloading mechanism
- Domains with several representations

Algorithmic

Internal algorithms

External software

- Symmetrica
- ► Ircalc
- gordan

Advanced demos

- Plethysms and other operators
- ► Hall-Littlewood, Macdonald
- ► LLT

What's wrong?

With symmetric functions in MuPAD-combinat

- Few users (ecological niche?)
- Very few contributers of new algorithmic (technological barrier)
- Remaining ACE / Lascoux algorithmic to be ported
- ▶ Too monolithic (lazier definitions, plug-in mechanisms)
- Speed?

With MuPAD-combinat

- ▶ Reaching the complexity limits of MuPAD
- ▶ MuPAD is not open source

Computing with symmetric functions???

What do you mean, really?

- What is it exactly that you want to compute?
- What does it mean to be efficient?

Examples

- Combinatorics :
 - very sparse symmetric functions of high degree
 - Symmetric series
- Symmetric polynomials
- Symmetric functions on alphabets
- Symmetric functions on concrete alphabets
- Schur-Schubert polynomials

More than one model for symmetric functions

- Sparse expanded representation
- ► Lazy (dense?) representation for series
 - Implementation by duality
 - Holonomic approach (Chyzac, Salvy, and co)
- Factorized / mixed expressions
- Straight line programs
- **•** ...

Which one(s) to implement?

FreeModules

- Encapsulation
- Internal data structure : kernel polynomials (variants possible)
- ► ~→ fast linear algebra (over kernel fields)
- Rankers (ranking/unranking of basis elements)
- ▶ Polynomials ~→ fast tensor products!

Overloading I

Conversions

- Fully centralized conversion graph
- ▶ Implicit conversions : canonical morphisms for all structures!
- Explicit conversions : aid to the user
- ▶ All domains are referenced there → no memory recollection

Overloading II

Overloading

- Operator : list of signatures
- Resolution: scan through signatures and find cheapest required conversions
- ➤ ¬¬ non natural liftings (natural ¬¬ strongly connected components?)
- \(\sim \) linear in the number of signatures
- ► Caching ~ fast later overloading resolution (one table lookup)
- ► Each modification invalidates the cache → bummer

It's good to be back at RISC!

Thanks Martin and Ralf!